Interesting Mathematical Stories

9.21.2024
Sohun

1　Convert repeating decimals to fractions

(1) Examples of repeating decimals

① 0.77777 ・・・ The number 7 is repeated after the 1st decimal place.
② 0.232323 ・・・ The number 23 is repeated after the 1st decimal place.
③ 0.245245245 ・・・ The number 245 is repeated after the 1st decimal place.
④ 0.6373737 ・・・ The number 37 is repeated after the 2nd decimal place.
⑤ 0.37545454 ・・・ The number 54 is repeated after the 3rd decimal place.
⑥ 0.9253253253 ・・・ The number 253 is repeated after the 2nd decimal place.
⑦ 0.36592592592 ・・・ The number 592 is repeated after the 3rd decimal place.

(2) How to express repeating decimals

① $0.77777\cdots = 0.\dot{7}$　② $0.232323\cdots = 0.\dot{2}\dot{3}$　③ $0.245245245\cdots = 0.\dot{2}4\dot{5}$

④ $0.6373737\cdots = 0.6\dot{3}\dot{7}$　⑤ $0.37545454\cdots = 0.37\dot{5}\dot{4}$

⑥ $0.9253253253\cdots = 0.9\dot{2}5\dot{3}$　⑦ $0.36592592592\cdots = 0.36\dot{5}9\dot{2}$

※ Place " ・ " above the first and last digits of the repeating numbers. (If there is only one repeating digit , there will be only one " ・ ".)

(3) How to convert repeating decimals to fractions.

(a) When the digits after the 1st decimal place are repeated.

$$0.\dot{7} = \frac{7}{9},\quad 0.\dot{4} = \frac{4}{9} \qquad \frac{(\textit{one repeated number})}{9}$$

$$0.\dot{2}\dot{3} = \frac{23}{99},\quad 0.\dot{8}\dot{5} = \frac{85}{99} \qquad \frac{(\textit{two repeated numbers})}{99}$$

$$0.\dot{2}4\dot{5} = \frac{245}{999},\quad 0.\dot{8}1\dot{3} = \frac{813}{999} \qquad \frac{(\textit{three repeated numbers})}{999}$$

numerator : Put repeated numbers

denominator : Put 9s in a row for each number of repeating numbers

〈Remarks〉 After converting the repeating decimals to a fraction ,
try calculating (numerator) ÷ (denominator) to convert it back to a decimal and verify that it matches the original repeating decimal.

Interesting Mathematical Stories

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1 Convert repeating decimals to fractions

（b） When two digits after the 2nd decimal place are repeated

$$0.6\dot{3}\dot{7}=\frac{631}{990}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{990}=\frac{637\text{-}6}{990}$$

$$0.8\dot{9}\dot{1}=\frac{883}{990}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{990}=\frac{891\text{-}8}{990}$$

（c） When three digits after the 2nd decimal place are repeated

$$0.9\dot{2}5\dot{3}=\frac{9244}{9990}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{9990}=\frac{9253\text{-}9}{9990}$$

$$0.2\dot{5}9\dot{7}=\frac{2595}{9990}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{9990}=\frac{2597\text{-}2}{9990}$$

（d） When two digits after the 3rd decimal place are repeated

$$0.37\dot{5}\dot{4}=\frac{3717}{9900}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{9900}=\frac{3754\text{-}37}{9900}$$

$$0.88\dot{2}\dot{9}=\frac{8741}{9900}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{9900}=\frac{8829\text{-}88}{9900}$$

（e） When three digits after the 3rd decimal place are repeated

$$0.89\dot{3}0\dot{7}=\frac{89218}{99900}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{99900}=\frac{89307\text{-}89}{99900}$$

$$0.36\dot{5}9\dot{2}=\frac{36556}{99900}\longleftarrow\frac{(\text{All digits after the decimal point on the left side})-(\text{Non-repeating numbers})}{99900}=\frac{36592\text{-}36}{99900}$$

（f） In general , how to convert the following repeating declmals ※ into fractions.

$$0.\,57\cdots48\dot{2}6\cdots1\dot{3}\ \text{---------}\ ※$$

$$\uparrow\uparrow\quad\uparrow\uparrow\uparrow\uparrow\quad\uparrow\uparrow$$

$$00\cdots0099\cdots99\ \text{--------}\ ☆$$

Like the ☆ above , 0 is assigned to no-repeating numbers and 9 is assigned to repeating numbers.

The numbers in the ☆ sequence created in this way are then arranged in reverse order to form the denominator.

Then , the numerator is the value obtained by subtracting no-repeating numbers（57・・・48）from the numbers（57・・・4826・・・13）after the decimal points of the ※.

Interesting Mathematical Stories

9.27.2024
Sohun

2 The thickness of a newspaper folded 50 times

Is it true that if you could fold a newspaper (thickness 0.1mm) 50 times , its thickness would far exceed the distance between the Earth and the Moon (384,440km)?

Fold it once , 0.1×2^{1} mm

Fold it twice , 0.1×2^{2} mm

Fold it 3 times , 0.1×2^{3} mm

⋮

Fold it 50 times , 0.1×2^{50} mm

If you can fold a newspaper (thickness 0.1mm) 50 times , its thickness will be 0.1×2^{50} mm.

$$\begin{aligned} & 0.1 \times 2^{50} \\ = & 0.1 \times (2^{10})^{5} \\ = & 0.1 \times (1024)^{5} \\ > & 0.1 \times (1000)^{5} \quad \cdots ※ \\ = & 0.1 \times (10^{3})^{5} \\ = & 0.1 \times 10^{15} \end{aligned}$$

0.1×2^{50} mm $>$ 0.1×10^{15} mm holds.

$$0.1 \times 10^{15} \text{ mm} = 0.1 \times 10^{9} \text{ km} = 1 \times 10^{8} \text{ km}$$
$$= 100,000,000 \text{ km}$$

Therefore ,

$$\underbrace{0.1 \times 2^{50} \text{ mm}}_{\text{The thickness of a newspaper folded 50 times}} > 100,000,000 \text{ km} > \underbrace{384,440 \text{km}}_{\text{Distance between the Earth and the Moon}}$$

Therefore ,
If you could fold a newspaper (thickness 0.1mm) 50 times , its thickness would far exceed the distance between the Earth and the Moon (384,440km).

(Remarks) When thinking about this story , the cleavor thing about it is that the calculations are simplified by transforming the equations and using inequalities in the ※ parts above.

(Consideration) So , how many times would a newspaper (thickness 0.1mm) have to be folded so that its thickness exceeds the distance (384,440km) between the Earth and the Moon ?

If a newspaper can be folded 40 times , its thickness will be 0.1×2^{40} mm.

$$\begin{aligned} & 0.1 \times 2^{40} \\ = & 0.1 \times (2^{10})^{4} \\ = & 0.1 \times (1024)^{4} \\ > & 0.1 \times (1000)^{4} \quad \cdots ※ \\ = & 0.1 \times (10^{3})^{4} \\ = & 0.1 \times 10^{12} \end{aligned}$$

2 The thickness of a newspaper folded 50 times

0.1×2^{40} mm $>$ 0.1×10^{12} mm holds.

0.1×10^{12} mm $=$ 0.1×10^{6} km $=$ 1×10^{5} km

$=100,000$ km

$\underbrace{0.1\times2^{40}\ \text{mm}}_{\text{The thickness of a newspaper folded 40 times}}>100,000\ \text{km}$ ・・・①

$\underbrace{0.1\times2^{41}\ \text{mm}}_{\text{The thickness of a newspaper folded 41 times}}>200,000\ \text{km}$ ・・・①$\times2$

$\underbrace{0.1\times2^{42}\ \text{mm}}_{\text{The thickness of a newspaper folded 42 times}}>400,000\ \text{km}$ ・・・①$\times2^{2}$

Therefore ,

$\underbrace{0.1\times2^{42}\ \text{mm}}_{\text{The thickness of a newspaper folded 42 times}}>400,000\ \text{km}>\underbrace{384,440\ \text{km}}_{\text{Distance between the Earth and the Moon}}$

Therefore .

If you could fold a newspaper (thickness 0.1mm) 42 times , its thickness would exceed the distance (384,440km) between the Earth and the Moon.

(Supplement)

If a newspaper can be folded 10 times , its thickness will be 0.1×2^{10} mm.

$$
\begin{aligned}
&0.1\times2^{10}\\
&=0.1\times(2^{10})\\
&=0.1\times(1024)\\
&>0.1\times(1000) \quad \cdots ※\\
&=0.1\times(10^{3})\\
&=0.1\times10^{3}
\end{aligned}
$$

0.1×2^{10} mm $>$ 0.1×10^{3} mm holds.

0.1×10^{3} mm $=100$ cm

$\underbrace{0.1\times2^{10}\ \text{mm}}_{\text{The thickness of a newspaper folded 10 times}}>100\ \text{cm}$ ・・・②

If a newspaper can be folded 20 times , its thickness will be 0.1×2^{20} mm.

$$
\begin{aligned}
&0.1\times2^{20}\\
&=0.1\times(2^{10})^{2}\\
&=0.1\times(1024)^{2}\\
&>0.1\times(1000)^{2} \quad \cdots ※\\
&=0.1\times(10^{3})^{2}\\
&=0.1\times10^{6}
\end{aligned}
$$

0.1×2^{20} mm $>$ 0.1×10^{6} mm holds.

0.1×10^{6} mm $=100$ m

$\underbrace{0.1\times2^{20}\ \text{mm}}_{\text{The thickness of a newspaper folded 20 times}}>100\ \text{m}$ ・・・③

Interesting Mathematical Stories

10.2.2024
Sohun

3 Calculate the number with root to three decimal places

（１） When the root symbol contains a single digit natural number

Find $\sqrt{7}$ to three decimal places.

$\boxed{2}$	7	*1st row*
$\boxed{2}$	4	
$4\boxed{6}$	3 0 0	*2nd row*
$\boxed{6}$	2 7 6	
$52\boxed{4}$	2 4 0 0	*3rd row*
$\boxed{4}$	2 0 9 6	
$528\boxed{5}$	3 0 4 0 0	*4th row*
$\boxed{5}$	2 6 4 2 5	

(Procedure)

Pay attension to the 1st row. Find the largest natural number whose square does not exceed the number 7 in tthe root. $2 \times 2=4$, so it's 2. Write the number 7 in the root and the 2 , 2 , and 4 of $2 \times 2 = 4$ as in the 1st row. Let's surround the both of natural number 2 , which does not exceed 7 when squared , with a rectangle.

Pay attension to the 2nd row. The sum of 2 and 2 in the 1st row , 4 , is written as in the 2nd row.The difference between 7 and 4 in the 1st row , 3 , is written as in the 2nd row. Then , add a 00 after the 3 to make it 300.

4 □×□ : Put the largest natural number that doesn't exceed 300 into □. However , two □ s must be the same numbers. Since $46 \times 6=276$, write 6 , 6 and 276 as in the 2nd row. Keep both 6 and 6 surrounded by a square.

Pay attension to the 3rd row. The sum of 46 and 6 in the 2nd row , 52 , is written as in the 3rd row. The difference between 300 and 276 in the 2nd row , 24 , is written as in the 3rd row . Then add a 00 after the 24 to make it 2400.

52 □×□ : Put the largest natural number that doesn't exceed 2400 into □. However , two □ s must be the same numbers. Since $524 \times 4=2096$, write 4 , 4 and 2096 as in the 3rd row. Keep both 4 and 4 surrounded by a square.

Pay attension to the 4th row. The sum of 524 and 4 in the 3rd row , 528 , is written as in the 4th row. The difference between 2400 and 2096 in the 3rd row , 304 , is written as in the 4th row . Then add a 00 after the 304 to make it 30400.

528 □×□ : Put the largest natural number that doesn't exceed 30400 into □. However , two □ s must be the same numbers. Since $5285 \times 5=26425$, write 5 , 5 and 26425 as in the 4th row. Keep both 5 and 5 surrounded by a square.

Therefore , take one number from each pair of natural numbers enclosed in a square in order.

$\sqrt{7} = 2.645$

(Remarks)

As mentioned above , calculations up to the 4th row were done to three decimal places.

Furthermore , if we calculate the 5th row and beyond in the same way , we can find the 4th , 5th , 6th decimal place , etc.

Interesting Mathematical Stories

10.1.2024
Sohun

3　Calculate the number with root to three decimal places

（2） When the root symbol contains two-digit natural number.

Find $\sqrt{51}$ to three decimal places.

$$
\begin{array}{r l r l}
 & \boxed{7} & 51 & \text{1st row} \\
 & \boxed{7} & 49 & \\ \hline
14 & \boxed{1} & 200 & \text{2nd row} \\
 & \boxed{1} & 141 & \\ \hline
142 & \boxed{4} & 5900 & \text{3rd row} \\
 & \boxed{4} & 5696 & \\ \hline
1428 & \boxed{1} & 20400 & \text{4th row} \\
 & \boxed{1} & 14281 &
\end{array}
$$

（Procedure）

Pay attension to the 1st row. Find the largest natural number whose square does not exceed the number 51 in tthe root. $7 \times 7=49$, so it's 7. Write the number 51 in the root and the 7 , 7 , and 49 of $7 \times 7 = 49$ as in the 1st row. Let's surround the both of natural number 7 , which does not exceed 51 when squared , with a rectangle.

Pay attension to the 2nd row. The sum of 7 and 7 in the 1st row , 14 , is written as in the 2nd row. The difference between 51 and 49 in the 1st row , 2 , is written as in the 2nd row. Then , add a 00 after the 2 to make it 200.

14 □×□ : Put the largest natural number that doesn't exceed 200 into □. However , two □ s must be the same numbers. Since $141 \times 1=141$, write 1 , 1 and 141 as in the 2nd row. Keep both 1 and 1 surrounded by a square.

Pay attension to the 3rd row. The sum of 141 and 1 in the 2nd row , 142 , is written as in the 3rd row. The difference between 200 and 141 in the 2nd row , 59 , is written as in the 3rd row . Then add a 00 after the 59 to make it 5900.

142 □×□ : Put the largest natural number that doesn't exceed 5900 into □. However , two □ s must be the same numbers. Since $1424 \times 4=5696$, write 4 , 4 and 5696 as in the 3rd row. Keep both 4 and 4 surrounded by a square.

Pay attension to the 4th row. The sum of 1424 and 4 in the 3rd row , 1428 , is written as in the 4th row. The difference between 5900 and 5696 in the 3rd row , 204 , is written as in the 4th row. Then add a 00 after the 204 to make it 20400.

1428 □×□ : Put the largest natural number that doesn't exceed 20400 into □. However , two □ s must be the same numbers. Since $14281 \times 1=14281$, write 1 , 1 and 14281 as in the 4th row. Keep both 1 and 1 surrounded by a square.

Therefore , take one number from each pair of natural numbers enclosed in a square in order.

$\sqrt{51} = 7.141$

（Remarks）

As mentioned above , calculations up to the 4th row were done to three decimal places.

Furthermore , if we calculate the 5th row and beyond in the same way , we can find the 4th , 5th , 6th decimal place , etc.

Interesting Mathematical Stories

10.3.2024
Sohun

3 Calculate the number with root to three decimal places

（３） When the root symbol contains three-digit natural number

Consider the numbers in pairs starting from the right.

Find $\sqrt{732}$ to three decimal places.

$\boxed{2}$	7 3 2	1st row
$\boxed{2}$	4	
4 $\boxed{7}$	3 3 2	2nd row
$\boxed{7}$	3 2 9	
5 4 $\boxed{0}$	3 0 0	3rd row
$\boxed{0}$	0	
5 4 0 $\boxed{5}$	3 0 0 0 0	4th row
$\boxed{5}$	2 7 0 2 5	
5 4 1 0 $\boxed{5}$	2 9 7 5 0 0	5th row
$\boxed{5}$	2 7 0 5 2 5	

(Procedure)

Pay attension to the 1st row. Find the largest natural number whose square does not exceed the number 7 that is the hundred digit in tthe root. 2 × 2=4 , so it's 2. Write the number 732 in the root and the 2 , 2 , and 4 of 2 × 2 = 4 as in the 1st row. Let's surround the both of natural number 2 , which does not exceed 7 when squared , with a rectangle.

Pay attension to the 2nd row. The sum of 2 and 2 in the 1st row , 4 , is written as in the 2nd row. The difference between 7 and 4 in the 1st row , 3 , is written as in the 2nd row. Take the number 32 in the root and put it after the 3 to make 332.

4 □×□ : Put the largest natural number that doesn't exceed 332 into □. However , two □ s must be the same numbers. Since 47 × 7=329 , write 7 , 7 and 329 as in the 2nd row. Keep both 7 and 7 surrounded by a square.

Pay attension to the 3rd row. The sum of 47 and 7 in the 2nd row , 54 , is written as in the 3rd row. The difference between 332 and 329 in the 2nd row , 3 , is written as in the 3rd row . Then add a 00 after the 3 to make it 300.

54 □×□ : Put the largest natural number that doesn't exceed 300 into □. However , two □ s must be the same numbers. Since 540 × 0=0 , write 0 , 0 and 0 as in the 3rd row. Keep both 0 and 0 surrounded by a square.

Pay attension to the 4th row. The sum of 540 and 0 in the 3rd row , 540 , is written as in the 4th row. The difference between 300 and 0 in the 3rd row , 300 , is written as in the 4th row. Then add a 00 after the 300 to make it 30000.

540 □×□ : Put the largest natural number that doesn't exceed 30000 into □. However , two □ s must be the same numbers. Since 5405 × 5=27025 , write 5 , 5 and 27025 as in the 4th row. Keep both 5 and 5 surrounded by a square.

Pay attension to the 5th row. The sum of 5405 and 5 in the 4th row , 5410 , is written as in the 5th row. The difference between 30000 and 27025 in the 4rd row , 2975 , is written as in the 5th row. Then add a 00 after the 2975 to make it 297500.

5410 □×□ : Put the largest natural number that doesn't exceed 297500 into □. However , two □ s must be the same numbers. Since 54105 × 5=270525 , write 5 , 5 and 270525 as in the 5th row. Keep both 5 and 5 surrounded by a square.

Therefore , take one number from each pair of natural numbers enclosed in a square in order.

$\sqrt{732} = 27.055$

(Remarks)

As mentioned above , calculations up to the 5th row were done to three decimal places.
Furthermore , if we calculate the 6th row and beyond in the same way , we can find the 4th , 5th , 6th decimal place , etc.

Interesting Mathematical Stories

10.4.2024
Sohun

3 Calculate the number with root to three decimal places

Consider the numbers in pairs starting from the right.

（4） When the root symbol contains four-digit natural number

Find $\sqrt{5273}$ to three decimal places.

$$
\begin{array}{r r l}
\boxed{7} & 5\,2\,7\,3 & \\
\boxed{7} & 4\,9 & \text{1st row}\\ \hline
1\,4\,\boxed{2} & 3\,7\,3 & \\
\boxed{2} & 2\,8\,4 & \text{2nd row}\\ \hline
1\,4\,4\,\boxed{6} & 8\,9\,0\,0 & \\
\boxed{6} & 8\,6\,7\,6 & \text{3rd row}\\ \hline
1\,4\,5\,2\,\boxed{1} & 2\,2\,4\,0\,0 & \\
\boxed{1} & 1\,4\,5\,2\,1 & \text{4th row}\\ \hline
1\,4\,5\,2\,2\,\boxed{5} & 7\,8\,7\,9\,0\,0 & \\
\boxed{5} & 7\,2\,6\,1\,2\,5 & \text{5th row}
\end{array}
$$

(Procedure)

Pay attension to the 1st row. Find the largest natural number whose square does not exceed the number 52 that is the thousands digit and hundreds digit in tthe root. 7 × 7=49 , so it's 7. Write the number 5273 in the root and the 7 , 7 , and 49 of 7 × 7 = 49 as in the 1st row. Let's surround the both of natural number 7 , which does not exceed 52 when squared , with a rectangle.

Pay attension to the 2nd row. The sum of 7 and 7 in the 1st row , 14 , is written as in the 2nd row. The difference between 52 and 49 in the 1st row , 3 , is written as in the 2nd row. Take the number 73 in the root and put it after the 3 to make 373.

14 □×□ : Put the largest natural number that doesn't exceed 373 into □. However , two □ s must be the same numbers. Since 142 × 2=284 , write 2 , 2 and 284 as in the 2nd row. Keep both 2 and 2 surrounded by a square.

Pay attension to the 3rd row. The sum of 142 and 2 in the 2nd row , 144 , is written as in the 3rd row. The difference between 373 and 284 in the 2nd row , 89 , is written as in the 3rd row . Then add a 00 after the 89 to make it 8900.

144 □×□ : Put the largest natural number that doesn't exceed 8900 into □. However , the two □ s must be the same numbers. Since 1446 × 6=8676 , write 6 , 6 and 8676 as in the 3rd row. Keep both 6 and 6 surrounded by a square.

Pay attension to the 4th row. The sum of 1446 and 6 in the 3rd row , 1452 , is written as in the 4th row. The difference between 8900 and 8676 in the 3rd row , 224 , is written as in the 4th row. Then add a 00 after the 224 to make it 22400.

1452 □×□ : Put the largest natural number that doesn't exceed 22400 into □. However , the two □ s must be the same numbers. Since 14521 × 1=14521 , write 1 , 1 and 14521 as in the 4th row. Keep both 1 and 1 surrounded by a square.

Pay attension to the 5th row. The sum of 14521 and 1 in the 4th row , 14522 , is written as in the 5th row. The difference between 22400 and 14521 in the 4rd row , 7879 , is written as in the 5th row. Then add a 00 after the 7879 to make it 787900.

14522 □×□ : Put the largest natural number that doesn't exceed 787900 into □. However , the two □ s must be the same numbers. Since 145225 × 5=726125 , write 5 , 5 and 726125 as in the 5th row. Keep both 5 and 5 surrounded by a square.

Therefore , take one number from each pair of natural numbers enclosed in a square in order.

$\sqrt{5273} = 72.615$

(Remarks)

As mentioned above , calculations up to the 5th row were done to three decimal places.

Furthermore , if we calculate the 6th row and beyond in the same way , we can find the 4th , 5th , 6th decimal place , etc.

Interesting Mathematical Stories

10.5.2024
Sohun

3 Calculate the number with root to three decimal places

Consider the numbers in pairs starting from the right.

(5) When the root symbol contains five-digit natural number

Find $\sqrt{39851}$ to three decimal places.

$\boxed{1}$	3 9 8 5 1	1st row
$\boxed{1}$	1	
2 $\boxed{9}$	2 9 8	2nd row
$\boxed{9}$	2 6 1	
3 8 $\boxed{9}$	3 7 5 1	3rd row
$\boxed{9}$	3 5 0 1	
3 9 8 $\boxed{6}$	2 5 0 0 0	4th row
$\boxed{6}$	2 3 9 1 6	
3 9 9 2 $\boxed{2}$	1 0 8 4 0 0	5th row
$\boxed{2}$	7 9 8 4 4	
3 9 9 2 4 $\boxed{7}$	2 8 5 5 6 0 0	6th row
$\boxed{7}$	2 7 9 4 7 2 9	

(Procedure)

Pay attension to the 1st row. Find the largest natural number whose square does not exceed the number 3 that is the ten thousands digit in tthe root. 1 × 1=1 , so it's 1. Write the number 39851 in the root and the 1 , 1 , and 1 of 1 × 1 = 1 as in the 1st row. Let's surround the both of natural number 1 , which does not exceed 3 when squared , with a rectangle.

Pay attension to the 2nd row. The sum of 1 and 1 in the 1st row , 2 , is written as in the 2nd row. The difference between 3 and 1 in the 1st row , 2 , is written as in the 2nd row. Take the number 98 in the root and put it after the 2 to make 298.

2 □×□ : Put the largest natural number that doesn't exceed 298 into □. However , the two □ s must be the same numbers. Since 29 × 9=261 , write 9 , 9 and 261 as in the 2nd row. Keep both 9 and 9 surrounded by a square.

Pay attension to the 3rd row. The sum of 29 and 9 in the 2nd row , 38 , is written as in the 3rd row. The difference between 298 and 261 in the 2nd row , 37 , is written as in the 3rd row . Take the number 51 in the root and put it after the 37 to make it 3751.

38 □×□ : Put the largest natural number that doesn't exceed 3751 into □. However , the two □ s must be the same numbers. Since 389 × 9=3501 , write 9 , 9 and 3501 as in the 3rd row. Keep both 9 and 9 surrounded by a square.

Pay attension to the 4th row. The sum of 389 and 9 in the 3rd row , 398 , is written as in the 4th row. The difference between 3751 and 3501 in the 3rd row , 250 , is written as in the 4th row. Then add a 00 after the 250 to make it 25000.

398 □×□ : Put the largest natural number that doesn't exceed 25000 into □. However , the two □ s must be the same numbers. Since 3986 × 6=23916 , write 6 , 6 and 23916 as in the 4th row. Keep both 6 and 6 surrounded by a square.

Pay attension to the 5th row. The sum of 3986 and 6 in the 4th row , 3992 , is written as in the 5th row. The difference between 25000 and 23916 in the 4rd row , 1084 , is written as in the 5th row. Then add a 00 after the 1084 to make it 108400.

3992 □×□ : Put the largest natural number that doesn't exceed 108400 into □. However , the two □ s must be the same numbers. Since 39922 × 2=79844 , write 2 , 2 and 79844 as in the 5th row. Keep both 2 and 2 surrounded by a square.

Pay attension to the 6th row. The sum of 39922 and 2 in the 5th row , 39924 , is written as in the 6th row. The difference between 108400 and 79844 in the 5rd row , 28556 , is written as in the 6th row. Then add a 00 after the 28556 to make it 2855600.

39924 □×□ : Put the largest natural number that doesn't exceed 2855600 into □. However , the two □ s must be the same numbers. Since 399247 × 7=2794729 , write 7 , 7 and 2794729 as in the 6th row. Keep both 7 and 7 surrounded by a square.

Therefore , take one number from each pair of natural numbers enclosed in a square in order.

$\sqrt{39851} = 199.627$

（Remarks）

As mentioned above , calculations up to the 6th row were done to three decimal places.

Furthermore , if we calculate the 7th row and beyond in the same way , we can find the 4th , 5th , 6th decimal place , etc.

Interesting Mathematical Stories

10.7.2024
Sohun

4 Multiplication of two-digit integer with the same tens digit

(1) $13\times18=(13+8)\times\underline{10}+\underline{3\times8}=210+24=234$

Multiplication of 1st digits

(2) $36\times39=(36+9)\times\underline{30}+\underline{6\times9}=1350+54=1404$

Multiplication of 1st digits

(3) $52\times57=(52+7)\times\underline{50}+\underline{2\times7}=2950+14=2964$

Multiplication of 1st digits

(4) $71\times74=(71+4)\times\underline{70}+\underline{1\times4}=5250+4=5254$

Multiplication of 1st digits

(5) $94\times96=(94+6)\times\underline{90}+\underline{4\times6}=9000+24=9024$

Multiplication of 1st digits

(6) $28\times22=(28+2)\times\underline{20}+\underline{8\times2}=600+16=616$

Multiplication of 1st digits

In general ,

Let an integer whose tens digit is a and whose ones digit is b be represented as ab.
Let an integer whose tens digit is a and whose ones digit is c be represented as ac.

$$ab\times ac=(\overline{10a+b})\times(10a+c)=(\overline{10a+b}+c)\times10a+bc$$

Multiplication of 1st digits

$$=100a^2+10ab+10ac+bc \quad \text{----------} ①$$

On the other hand , when (10a+b) (10a+c) is expanded , it becoms $100a^2+10ab+10ac+bc$, which can be confirmed to be equal to ①.

Interesting Mathematical Stories

10.26.2024
Sohun

5 How to find multiples

(1) Multiple of 2
The ones digit is an even number.

(2) Multiple of 3
A number whose sum of each digit is a multiple of 3.

(3) Multiple of 4
The last two digits are a multiple of 4.

(4) Multiple of 5
The ones digit is 0 or 5.

(5) Multiple of 6
A number whose ones digit is even and whose sum of each digit is a multiple of 3.

(6) Multiple of 7
Divide the digits into groups of three , starting from the ones digit , and add or subtract them altermately to get a number that is a multiple of 7.

(Example) 2 933 (933 − 2 = 931 is a multiple of 7)

70 280 (280 − 70 = 210 is a multiple of 7)

582 092 (92 − 582 = −490 is a multiple of 7)

1 003 093 (93 − 3 + 1 = 91 is a multiple of 7)

(7) Multiple of 8
The last three digits are a multiple of 8.

(8) Multiple of 9
A number whose sum of each digit is a multiple of 9.

(9) Multiple of 10
The ones digit is 0.

(10) Multiple of 11
Divide the digits into groups of three , starting from the ones digit , and add or subtract them altermately to get a number that is a multiple of 11.
(Refer to a multiple of 7 above)

(11) Multiple of 12
The last two digits are a multiple of 4 and the sum of each digit is a multiple of 3.

(12) Multiple of 13
Divide the digits into groups of three , starting from the ones digit , and add or subtract them altermately to get a number that is a multiple of 13.
(Refer to a multiple of 7 above)

< Remarks > Classification by similar finding

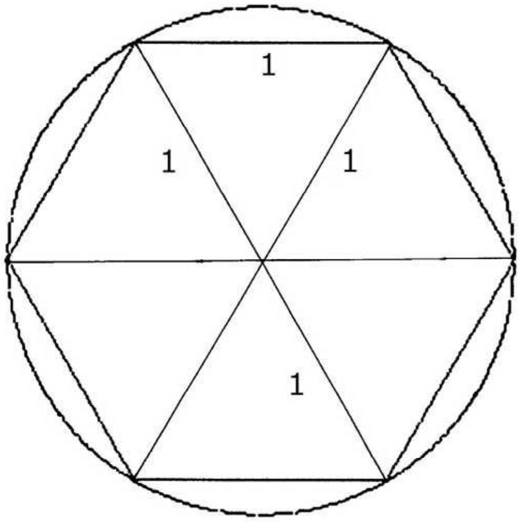
- Multiple of 3 / Multiple of 9
- Multiple of 4 / Multiple of 8
- Multiple of 7 / Multiple of 11 / Multiple of 13
- Multiple of 2 / Multiple of 5 / Multiple of 10
- Multiple of 6 / Multiple of 12

Interesting Mathematical Stories

10.29.2024
Sohun

6 Pi is greater than 3

（1） Consider a circle of radius 1 and a regular hexagon inscribed in it.



As shown in the diagram above , draw three diagonals of a regular hexagon that pass through the center of the circle.
A regular hexagon is devided into six equilateral triangles with sides of length 1.

（2） From the diagram avove , the following inequality clearly holds.
(Circumference of a circle with radius 1)＞(Perimeter of a regular hexagon inscribed in a circle of radius 1) --- ①

（3） Circumference of a circle with radius 1 = 2pi --- ② (Circumference = Diameter × pi)

Perimeter of a regular hexagon inscribed in a circle of radius 1 = 6 --- ③ (From diagram above)

（4） Subsitute ② and ③ into the inequality in ①.
2pi ＞ 6 --- ④

（5） Divide both sides of ④ by 2.
pi ＞ 3

（6） Therefore , we know that pi is greater than 3.

Interesting Mathematical Stories

10.26.2024
Sohun

7 The rabbit can never overtake the turtle

The rabbit's speed is twice as fast as the the turtle. For example , let the rabbit's speed be 10 m/s and the turtle's speed be 5 m/s.

The turtle starts at the same time as the rabbit , 10 meters ahead of the starting line.

(Step 1)

First , the rabbit moves to the turtle's position , 10 meters ahead of the starting line.

At this point , the turtle has moved 10 + 5 meters ahead of the starting line.

(Step 2)

Next , the rabbit moves to the turtle's position , 10+5 meters ahead of the starting line.

At this point , the turtle has moved 10 + 5 + 2.5 meters ahead of the starting line.

(Step 3)

Furthermore , the rabbit moves to the turtle's position , 10+5+2.5 meters ahead of the starting line.

At this point , the turtle has moved 10 + 5 + 2.5+1.25 meters ahead of the starting line.

(Step 4)

Furthermore , the rabbit moves to the turtle's position , 10+5+2.5+1.25 meters ahead of the starting line.

At this point , the turtle has moved 10 + 5 + 2.5+1.25+0.625 meters ahead of the starting line.

(Step 5)

Furthermore , the rabbit moves to the turtle's position , 10+5+2.5+1.25+0.625 meters ahead of the starting line.

At this point , the turtle has moved 10 + 5 + 2.5+1.25+0.625+0.3125 meters ahead of the starting line.

If we think about it this way , it seems that the rabbit will never be able to overtake the turtle . But in reality , the rabbit overtakes the turtle . Where are we going wrong ?

The time it takes for Step 1 to move is 1 second , since the rabbit only moves 10m.
The time it takes for Step 2 to move is 0.5 second , since the rabbit only moves 5m.
The time it takes for Step 3 to move is 0.25 second , since the rabbit only moves 2.5m.
The time it takes for Step 4 to move is 0.125 second , since the rabbit only moves 1.25m.
The time it takes for Step 5 to move is 0.0625 second , since the rabbit only moves 0.625m.

In other words , if we consider the above , the time would be : 1+0.5+0.25+0.125+0.0625+・・・ ①

① is the sum of an infinite geometric series with first term 1 and common ratio 1/2.

Therefore , from the formula for the sum of a infinite geometric series ,

$$①=\frac{1}{1-\frac{1}{2}}=2$$

It turns out this way of thinking lasts less than two seconds. Instead of saying "The rabbit can never overtake the turtle" , it becomes "The rabbit can not overtake the turtle in less than two seconds".

(Remarks)

Let t seconds be the time that has elapsed since the start.
The turtle's position from the starting line after t seconds is 10+5t (m)
The rabbit's position from the starting line after t seconds is 10t (m)
10+5t=10t , so t=2.
Therefore , the rabbit catches up with the turtle in 2 seconds.

Interesting Mathematical Stories

11.5.2024
Sohun

8 Look for the perfect number , not a brute force approach

The positive divisors of 6 are 1 , 2 , 3 , and 6.
The positive divisors of 28 are 1 , 2 , 4 , 7 , 14 , and 28.
A natural number whose sum of all its positive divisors , excluding itself , is equal to itself is called a "perfect number".
For example , the positive divisors of 6 are 1 , 2 , 3 , and 6 , so the sum of the positive divisors excluding 6 is 1+2+3=6. Therefore , 6 is a perfect number.
Also , the positive divisors of 28 are 1 , 2 , 4 , 7 , 14 and 28 , so the sum of the positive divisors excluding 28 is 1+2+4+7+14=28. Therefore , 28 is a perfect number.
To find a perfect number , it would take a considerable amount of time to search through all the natural numbers starting from 1 , whose sum of all positive divisors excluding the number itself is equal to the number itself , even using a computer.
By the way , when n is a natural number , and (2^n-1) is a prime number , we know that $2^{n-1}(2^n-1)$ is a perfect number. A prime number is a natural number greater than or equal to 2 that has only 1 and itself as positive divisors. For example , the positive divisors of 2 are 1 and 2. The positive sivisors of 3 are 1 and 3. The positive divisors of 5 are 1 and 5. The positive divisors of 7 are 1 and 7. The positive divisors of 11 are 1 and 11. The above numbers 2 , 3 , 5 , 7 , 11 are prime numbers.
However , as to whether all perfect numbers can be expressed in form $2^{n-1}(2^n-1)$, it has been proven that this is true for all even perfect numbers. Additionally , no odd perfect numbers have yet been found.
By using this , it is possible to find the 8th smallest perfect number , <u>2,305,843,008,139,950,000</u> , in a relatively short amount of time using a personal computer.
The table below shows the results of finding perfect numbers using the "VBA" macro in the spread sheet software "Excel". We found up to 8 perfect numbers in ascending order. Even the 6th perfect number is over 100 million.

① When n=2 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=6$ is a perfect number.

② When n=3 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=28$ is a perfect number.

③ When n=5 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=496$ is a perfect number.

④ When n=7 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=8{,}128$ is a perfect number.

⑤ When n=13 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=33{,}550{,}336$ is a perfect number.

⑥ When n=17 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=8{,}589{,}869{,}056$ is a perfect number.

⑦ When n=19 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=137{,}438{,}691{,}328$
is a perfect number.

⑧ When n=31 , (2^n-1) is a prime number.
$2^{n-1}(2^n-1)=2{,}305{,}843{,}008{,}139{,}950{,}000$
is a perfect number.

n	2^n-1	Prime ?	$2^{n-1}(2^n-1)$	Perfect ?
2	3	Prime	6	Perfect
3	7	Prime	28	Perfect
4	15		120	
5	31	Prime	496	Perfect
6	63		2016	
7	127	Prime	8128	Perfect
8	255		32640	
9	511		130816	
10	1023		523776	
11	2047		2096128	
12	4095		8386560	
13	8191	Prime	33550336	Perfect
14	16383		134209536	
15	32767		536854528	
16	65535		2147450880	
17	131071	Prime	8589869056	Perfect
18	262143		34359607296	
19	524287	Prime	137438691328	Perfect
20	1048575		549755289600	
21	2097151		2199022206976	
22	4194303		8796090925056	
23	8388607		35184367894528	
24	16777215		140737479966720	
25	33554431		562949936644096	
26	67108863		2251799780130820	
27	134217727		9007199187632130	
28	268435455		36028796884746200	
29	536870911		144115187807420000	
30	1073741823		576460751766553000	
31	2147483647	Prime	2305843008139950000	Perfect

Interesting Mathematical Stories

11.8.2024
Sohun

9 e^{π} is greater than π^{e}

$e = 2.71828\cdots$ is a non-repeating infinitesimal (irrational number) and a transcendental number.
$\pi = 3.14159\cdots$ is also a non-repeating infinitesimal (irrational number) and a transcendental number.

A transcendental number is a number that is not a solution to the equation $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$ (Here , n is a natural number and a_n is an interger).

$$\left.\begin{aligned} e^{\pi} &= e^{\frac{1}{e} \times \pi e} \\ \pi^{e} &= \pi^{\frac{1}{\pi} \times \pi e} \end{aligned}\right\}$$

After transforming it like this ,

we compare the sizes of $e^{\frac{1}{e}}$ *and* $\pi^{\frac{1}{\pi}}$

Consider the graph of $y = x^{\frac{1}{x}}$ $(x>0)$

Take the natural logarithm of both sides of $y = x^{\frac{1}{x}}$

$$\log_e y = \log_e x^{\frac{1}{x}}$$

$$\log_e y = \frac{1}{x} \log_e x \quad \cdots \quad ①$$

Differentiate both sides of ① with respect to x.

$$\frac{dy}{dx} \times \frac{1}{y} = -\frac{1}{x^2} \log_e x + \frac{1}{x} \times \frac{1}{x}$$

$$\frac{dy}{dx} = y\left(-\frac{1}{x^2} \log_e x + \frac{1}{x} \times \frac{1}{x}\right)$$

Let $\frac{dy}{dx} = \frac{1}{x^2} \times x^{\frac{1}{x}} (1 - \log_e x) = 0$

x=e and create an increase / decrease table.

x	0		e		π	
$\frac{dy}{dx}$		+	0	−	−	−
y		↗	$e^{\frac{1}{e}}$	↘	$\pi^{\frac{1}{\pi}}$	

Interesting Mathematical Stories

11.8.2024
Sohun

9 e^{π} is greater than π^{e}

Therefore , $e^{\frac{1}{e}} > \pi^{\frac{1}{\pi}}$ ---②

Raise both sides of ② to the power of πe .

$$\left(e^{\frac{1}{e}}\right)^{\pi e} > \left(\pi^{\frac{1}{\pi}}\right)^{\pi e}$$

$$e^{\pi} > \pi^{e}$$

Therefoe , e^{π} is greater than π^{e} .

Interesting Mathematical Stories

11.5.2024
Sohun

１０　Scatter sesame seeds to find an approximation of pi

Draw a square and a circle inscribed in it.
Scatter sesame seeds randomly on top.
In this case , the approximate value of pi can be found using the following formula.

pi ＝ (Number of sesame seeds in a circle) ÷ (Number of sesame seeds in a square) × 4

Now , let's think about why this formula can give us an approximation of pi.
Let the radius of the circle be a.
The length of one side of the square is 2a.
The area of the circle is πa^2.
The area of the square is $4a^2$.

Let the number of the sesame seeds in the circle be n.
Let the number of the sesame seeds in the square be N.

In this case ,
(Area of the circle) : (Area of the square)
＝(Number of sesame seeds in tne circle) : (Number of sesame seeds in tne square)

$\pi a^2 : 4a^2 = n : N$ holds

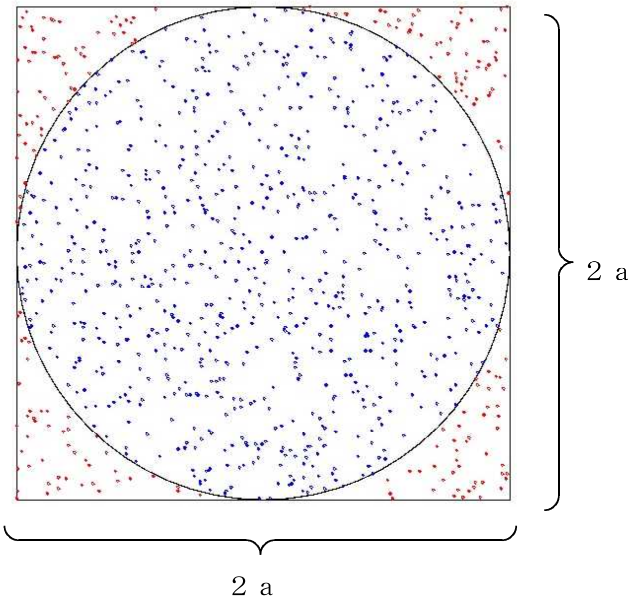
Therefore , $\pi a^2 N = 4a^2 n$

$$\pi = \frac{4a^2 n}{a^2 N}$$

$$= \frac{4n}{N}$$

$$= n \div N \times 4$$

Therefore ,
pi ＝ (Number of the sesame seeds in the circle) ÷ (Number of the sesame seeds in the square) × 4



The diagram on the left shows a square with sides of length 2a and a circle inscribed in it with radius a.
The dots represent sesame seeds.
The number of sesame seeds in the square , N , was 1000.
The number of sesame seeds in the circle, n , was 783.

Use the formula above to calculate an approximation of pi.

pi ＝ 783 ÷ 1000 × 4
＝ 3.132

Interesting Mathematical Stories

11.14.2024
Sohun

1 1　Scatter 10 yen coins to find an approximation of pi

Draw many parallel lines horizontally , spaced equally apart.
Furthermore , draw many parallel lines vertically , spaced equally apart.
(However , both the horizontal and vertical paraiiel lines spacing is equal to the diameter of the 10 yen coin)

It is called a grid line.
(See the figure below)

Scatter 10 yen coins randomly on top of it.
In this case , the approximate value of pi can be found using the following formula.

pi = (Number of 10 yen coins overlapping with the grid points)
÷ (Total number of 10 yen coins scattered) × 4
(The intersections of the horizontal and vertical parallel lines of the grid line are called grid points.)

So , why can we use this formula to find an approximation of pi ?
Let the diameter of a 10 yen coin be 2a.
The spacing between the parallel lines of the grid lines is 2a. (Because the spacing between them is equal to the diameter of a 10 yen coin.)

In the diagram below , the black lines represent the grid lines.
Draw a black dotted lines parallel to the center of the parallel grid lines.
In order for a 10 yen coin to overlap with grid point P in the diagram below , the center of the 10 yen coin must be within the inscribed circle in the red square. If the center of the 10 yen coin is inside the red square but outside the inscribed circle , grid point P and the 10 yen coin don't overlap.

In this case ,
(Area of the inscribed circle) : (Area of the square)
= (Number of centers of 10 yen coins in an inscribed circle)
: (Number of centers of 10 yen coins in a square) ・・・ ①　holds.

"Number of centers of 10 yen coins in an inscribed circle" and "Number of 10 yen coins whose center falls within the inscribed circle" have the same meaning.
"Number of centers of 10 yen coins in a square" and "Number of 10 yen coins whose center falls within the square" have the same meaning.

Let the number of centers of 10 yen coins in the inscribed circle be n.
Let the number of centers of 10 yen coins in the square be N.
The radius of the inscribed circle is a.
The length of one side of the square is 2a.
The area of the inscribed circle is πa^2.
The area of the square is $4a^2$.

Substitute into the proportional equation in ①.

$\pi a^2 : 4a^2 = n : N$　holds.

Therefore , $\pi a^2 N = 4a^2 n$

$$\pi = \frac{4a^2 n}{a^2 N}$$

$$= \frac{4n}{N}$$

$$= n \div N \times 4$$

Interesting Mathematical Stories

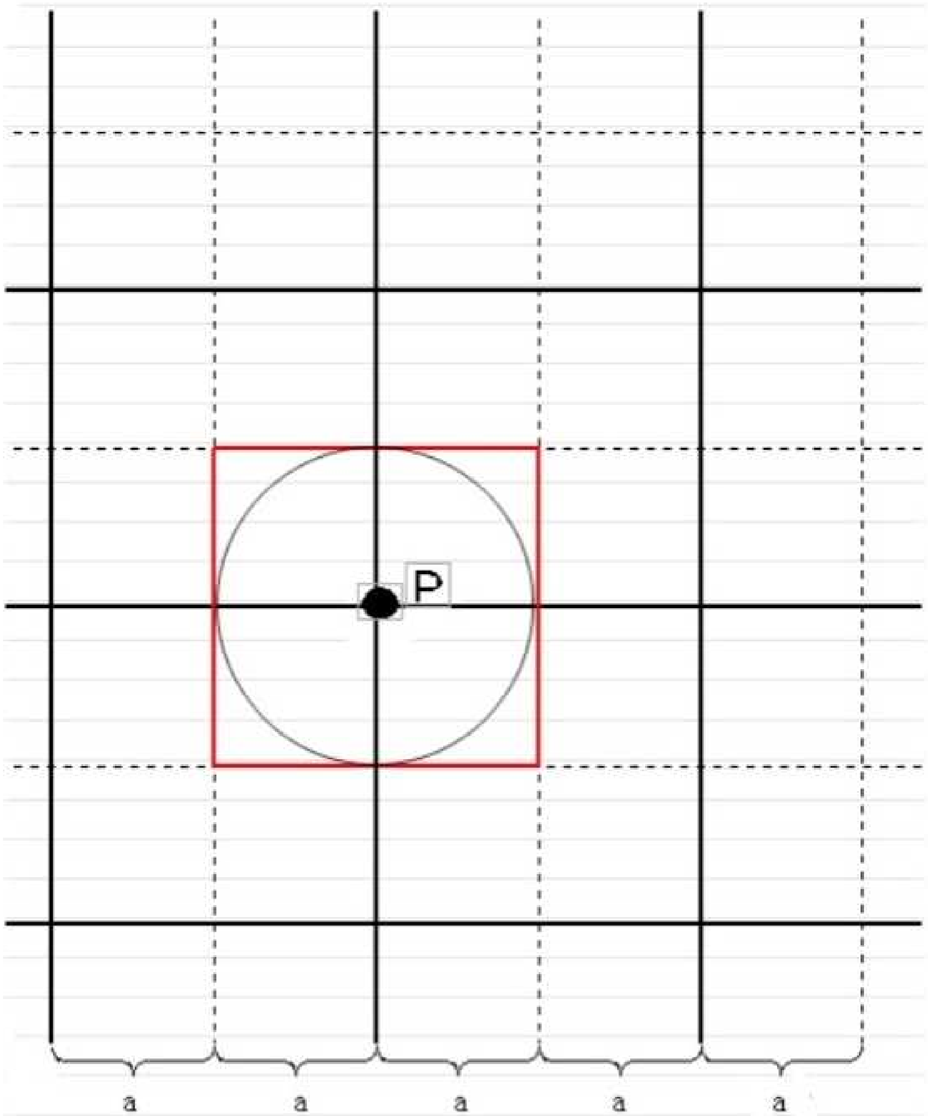
11.14.2024
Sohun

1 1　Scatter 10 yen coins to find an approximation of pi

Therefore ,
pi ＝(Number of the centers of 10 yen coins in the inscribed circle)
÷(Number of the centers of 10 yen coins in the square)× 4

In other words .
pi ＝(Number of 10 yen coins whose centers are in the inscribed circle)
÷(Number of 10 yen coins whose centers are in the square)× 4

That is ,
pi ＝ (Number of 10 yen coins overlapping with the grid points)
÷ (Total number of 10 yen coins scattered) × 4
holds.



Interesting Mathematical Stories

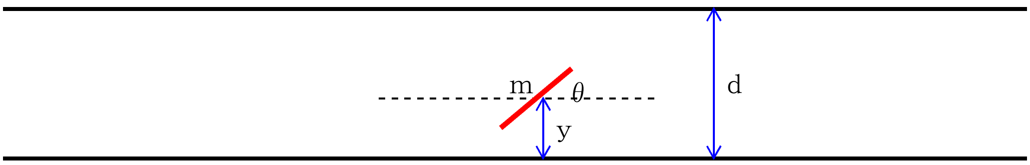
11.18.2024
Sohun

12 Scatter needles to find an approximation of pi

Draw many parallel lines at equal intervals.
Scatter needles of the same length randomly over it.
(However , the length of the needle should be half the distance between the parallel lines.)
In this case , the approximae value of pi can be found using the following formula.

pi = (Total number of needles scattered) ÷ (Number of needles overlapping with parallel lines)
(However , all needles that don't overlap with the parallel lines fall between the parallrel lines.)

So , why can we use this formula to find an approximation of pi ?
Let the length of the needle be m.
Let the distance between the parallel lines be d.
Let the angle between the scattered needle and the parallel line be θ .
Let the distance between the center of the scattered needle and the parallel line be y. (Here , y is the distance closer to the parallel line.)

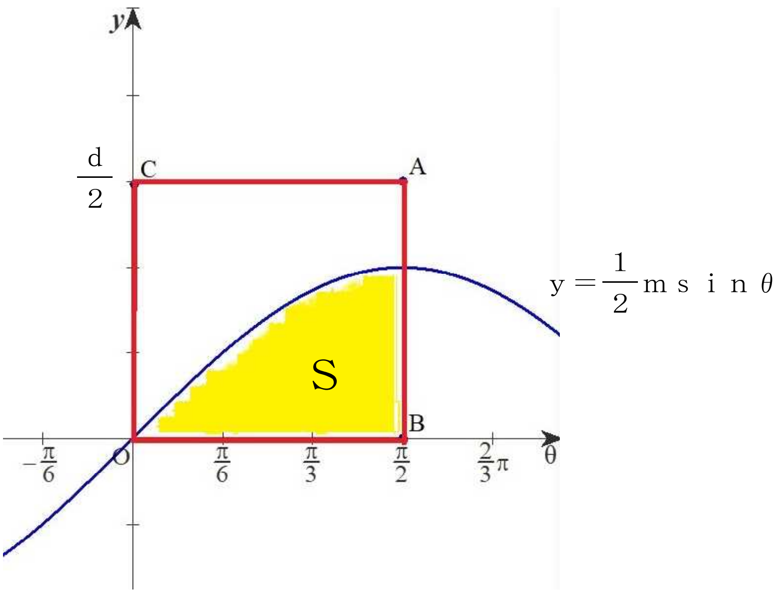


Black line : parallel lines
Red line : needle

When $0 \leqq y \leqq \frac{1}{2} m \sin\theta$, the scattered needles and the parallel lines overlap.

Here , $0 \leqq \theta \leqq \frac{1}{2}\pi$ 、 $0 \leqq y \leqq \frac{1}{2} d$ *are satisfied.*

Draw a graph of $y = \frac{1}{2} m \sin\theta$.



12 Scatter needles to find an approximation of pi

$$\frac{\textit{Yellow Area } S}{\textit{Area of } \square OBAC} = \textit{The percentage of scattered needles that overlap with parallel lines} \quad \cdots \text{①}$$

$$\textit{Area of } \square OBAC = \frac{\pi}{2} \times \frac{d}{2} = \frac{\pi d}{4} \quad \cdots \text{②}$$

$$\textit{Yellow area } S = \int_0^{\pi/2} \frac{1}{2} m \sin\theta \, d\theta$$

$$= \frac{1}{2} m \int_0^{\pi/2} \sin\theta \, d\theta$$

$$= \frac{1}{2} m \left[-\cos\theta\right]_0^{\pi/2}$$

$$= \frac{1}{2} m \left(-\cos\frac{\pi}{2} + \cos 0\right)$$

$$= \frac{1}{2} m (0 + 1)$$

$$= \frac{1}{2} m \quad \cdots \text{③}$$

Let the total number of needles scattered be N.
Let the number of needles overlapping with parallel lines be n.

Substitute ② , ③ , n , and N into ①.

$$\frac{\frac{1}{2}m}{\frac{\pi d}{4}} = \frac{n}{N} \quad \cdots \text{④}$$ holds.

Since the length m of the needle is half the distance d between the parallel lines , it can be expressed as d=2m.
Substitute d=2m into ④ and eliminate d.

$$\frac{\frac{1}{2}m}{\frac{\pi(2m)}{4}} = \frac{n}{N}$$

Tidy up the left side.

$$\frac{1}{\pi} = \frac{n}{N}$$

Therefore , $\pi = \dfrac{N}{n}$

Therefore ,
pi = (Total number of scattered needles) ÷ (Number of needles overlapping with parallel lines)

Interesting Mathematical Stories

5.14.2025
Sohun

１３ Find the sets of the sum of three-digit positive integers that satisfy the following conditions :

＜ Conditions ＞
（１） The sum is also a three-digit integer.
（２） Each digit in rhe number to be added consists of a different integer from 1 to 9.
（３） Each digit in rhe number to add consists of a different integer from 1 to 9.
（４） Each digit of rhe sum also consists of a different integer from 1 to 9.
（５） The nine integers in each digit of the number to be added , the number to add , and the sum are all different integers from 1 to 9.

（Ex 1） Number to be added 124
Number to add 659
Sum 783

（Ex 2） Number to be added 784
Number to add 152
Sum 936

I created a macro in Excel to find this.
The flow of the macro is as follows :
① Find the sums of the 9^6 ways from（111+111） to （999+999）.
② Among the sums obtained in ① , exclude those that have four digits.
③ Among the sums obtained in ② , exclude those where the tens or ones digit is 0.
④ Among the sums obtained in ③ , exclude those where the nine integers in the "number to be added" , "number to add" , and "sum" are all different are selected and displayed.

I searched using a laptop（NEC NX850/N）. In 2 minutes 25 seconds , all the matching set of three-digit integers were displayed.

Case Number	1	2	3	4	5	6	7	8	9	10
Number to be added	124	125	127	127	128	128	129	129	129	129
Number to add	659	739	359	368	367	439	357	438	654	735
Sum	783	864	486	495	495	567	486	567	783	864

Case Number	11	12	13	14	15	16	17	18	19	20
Number to be added	134	135	138	138	139	139	142	142	143	145
Number to add	658	729	429	654	428	725	596	695	586	692
Sum	792	864	567	792	567	864	738	837	729	837

Case Number	321	322	323	324	325	326	327	328	329	330
Number to be added	736	736	738	739	745	745	746	748	752	754
Number to add	218	245	216	125	218	236	235	215	184	182
Sum	954	981	954	864	963	981	981	963	936	936

Case Number	331	332	333	334	335	336				
Number to be added	762	763	782	782	783	784				
Number to add	183	182	154	163	162	152				
Sum	945	945	936	945	945	936				

336 sets of three-digit positive integers are displayed.
Half of these 336 sets are the same if you swap the numbers to be added and the numbers to add . Therefore , there are 168 sets of three-digit integers that satisfy the question , which is half of 336.